



The Space Congress® Proceedings

1965 (2nd) New Dimensions in Space
Technology

Apr 5th, 8:00 AM

A Signal Processing Technique for Space Object Identification

Lawrence A. Wan
Hughes Aircraft Company

Follow this and additional works at: <https://commons.erau.edu/space-congress-proceedings>

Scholarly Commons Citation

Wan, Lawrence A., "A Signal Processing Technique for Space Object Identification" (1965). *The Space Congress® Proceedings*. 3.

<https://commons.erau.edu/space-congress-proceedings/proceedings-1965-2nd/session-6/3>

This Event is brought to you for free and open access by the Conferences at Scholarly Commons. It has been accepted for inclusion in The Space Congress® Proceedings by an authorized administrator of Scholarly Commons. For more information, please contact commons@erau.edu.

EMBRY-RIDDLE
Aeronautical University™
SCHOLARLY COMMONS

A SIGNAL PROCESSING TECHNIQUE FOR SPACE OBJECT IDENTIFICATION

Lawrence A. Wan
Hughes Aircraft Company
Fullerton, California

Summary

This paper develops a radar system which is capable of measuring the lateral dimension of satellites. Such information contributes toward the identification of space objects and assessing their missions. The lateral dimension of a satellite can be inferred from the resolution of the angular separation of scatterers on that object.

Fundamentally, the technique employs doppler data processing in such a manner as to effectively synthesize a large radar antenna. A time invariant angle was formulated from the orbital trajectory and is the basic parameter which is measured to determine the spatial separation of scatterers on the satellite. This is a signal processing technique which in a certain sense may be viewed as the inverse of the familiar ground mapping "synthetic aperture". However, it is more complex in that unlike the ground-mapping aircraft, the dynamics of the space object are unknown or are statistical quantities. This analysis does present a radar system and a signal processing method which is capable of the extreme angular resolution requirement for high orbiting vehicles that would otherwise be beyond the course resolvability of conventional radar techniques.

Introduction

With the rapidly increasing number and varied deployment of artificial space satellites, the identification of such space objects in order to assess their missions is of scientific as well as military significance. Some of the information which must be acquired for mission assessment are the satellite's orbital parameters and a determination of its size to contribute toward its identification. The size of a space object may be inferred from the resolution of individual scatterers on the satellite. Such angular resolution by conventional radar techniques requires extremely large antenna apertures and short transmitted wavelengths. Thus in general, the high angular resolution which would be necessary to separate scatterers on the same object when coupled with the long satellite ranges is beyond the capabilities of normal radars. This inadequacy arises when relying on the relatively coarse antenna angular

resolution at such long ranges to separate the closely spaced scatterers. It suggests that rather than scatterers separation by direct antenna angular resolution, some other satellite parameter which can be extracted by appropriate signal processing instead, but will provide angular resolution indirectly, is left as the source of solution. Furthermore, signal parameter separation as a basis of satellite sizing, means that if $s_1(t, x)$ is the signal returned from a scatterer on the satellite, and $s_2(t, x)$ is the signal returned from a second scatterer, $s_1(t, x)$ and $s_2(t, x)$ must be distinguishable with respect to the parameter, x . A convenient mathematical measure of signal distinguishability, is the integrated absolute difference

$$\epsilon = \int_0^T |s_1(t, x) - s_2(t, x)|^2 dt \quad (1)$$

This difference should be large with respect to the parameter of interest in order to achieve signal separability. Further note that,

$$\epsilon = \int_0^T |s_1(t, x)|^2 dt + \int_0^T |s_2(t, x)|^2 dt - 2\text{Re} \int_0^T s_1(t, x) s_2^*(t, x) dt$$

in which the first two terms represent simply returned energies in the observation interval, T , and thus for comparable back-scatter cross-sections, only the last term determines the signals' separability. Therefore, to maximize the distinguishability of the two signals, implies that

$$\Delta(x) = \text{Re} \int_0^T s_1(t, x) s_2^*(t, x) dt \quad (2)$$

should be minimized with respect to x . In other words, $\Delta(x)$ is a measure of the resolvability of the parameter x , on the basis of signal separation. It thus, remains to establish the signal parameter which will provide satellite size information and derive the resolvability of this parameter in terms of the above expression. First in the procedure is to develop a reference coordinate system.

The Coordinate Reference

Consider the geocentric coordinate system shown in Figure 1. The satellite is at a distance $r(t)$ from the radar and $R(t)$ from the center of the earth. The radius of the earth is ρ . For illustration, two scatterers on the satellite are numbered #1 and #2. Thus, $r_1(t)$ and $r_2(t)$ are their respective distances from the radar. $R_1(t)$ and $R_2(t)$ are their respective distances from the earth center. Furthermore, let the satellite travel in a geocentric circular orbit with velocity, v . This approximation

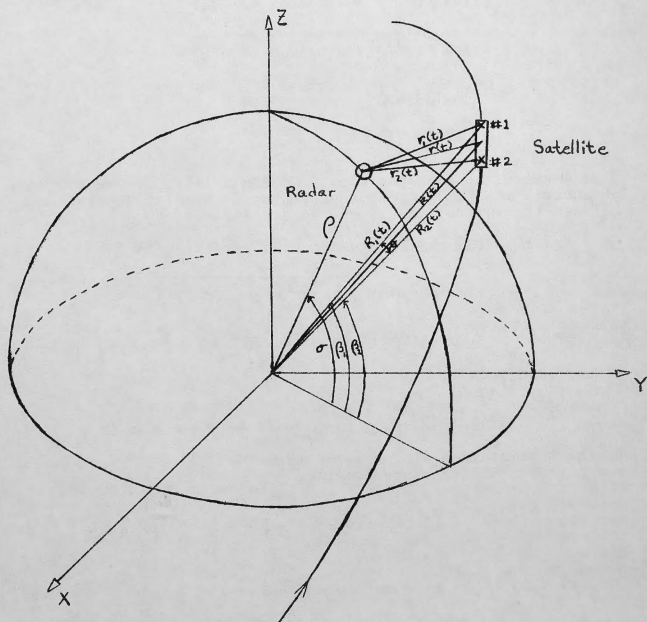


Figure I The Geocentric Coordinate Reference

is valid for contemporary artificial satellites, especially over short observation intervals T , of a few seconds. This can be readily verified by the list of present-day orbital parameters found in reference (1). It then follows from the geometry that $R(t) = R_1(t) = R_2(t) = R$, a constant. Furthermore,

$$r_1(t) = \sqrt{(R^2 + \rho^2) - 2\rho R \cos(\sigma - \beta_1)} \quad (3)$$

$$r_2(t) = \sqrt{(R^2 + \rho^2) - 2\rho R \cos(\sigma - \beta_2)}$$

where

$$\beta_1 = \frac{v}{R}t + \phi$$

$$\beta_2 = \frac{v}{R}t + \phi + \theta$$

Thus ϕ represents the position of the satellite on commencement of observation ($t=0$) and θ represents the angular separation between the two scatterers on the satellite.

Equation (3) can be expanded and rewritten as

$$\begin{aligned} r_1(t) &= (R^2 + \rho^2)^{\frac{1}{2}} \sqrt{1 - \frac{2\rho R}{R^2 + \rho^2} \cos(\sigma - \beta_1)} \\ &= (R^2 + \rho^2)^{\frac{1}{2}} \left[1 - q \cos(\sigma - \beta_1) - \frac{q^2}{4} \cos^2(\sigma - \beta_1) \right] \quad (4) \end{aligned}$$

since $q = \frac{2\rho R}{R^2 + \rho^2} \ll 1$.
Similarly, then

$$r_2(t) = (R^2 + \rho^2)^{\frac{1}{2}} \left[1 - q \cos(\sigma - \beta_2) - \frac{q^2}{4} \cos^2(\sigma - \beta_2) \right] \quad (5)$$

On the substitution of the expressions for β_1 and β_2 , the radar to scatterers ranges become,

$$r_1(t) = (R^2 + \rho^2)^{\frac{1}{2}} \left[1 - q \cos(\sigma - \frac{v}{R}t - \phi) - \frac{q^2}{4} \cos^2(\sigma - \frac{v}{R}t - \phi) \right] \quad (6)$$

and

$$r_2(t) = (R^2 + \rho^2)^{\frac{1}{2}} \left[1 - q \cos(\sigma - \frac{v}{R}t - \phi - \theta) - \frac{q^2}{4} \cos^2(\sigma - \frac{v}{R}t - \phi - \theta) \right] \quad (7)$$

The Resolution Function

Return now to equation (2) and consider the signals returned from these two scatterers. If the transmitted signal is $s(t) = a(t)\exp(j\omega t)$, where $a(t)$ is the modulation waveform and $\exp(j\omega_0 t)$ is the high frequency carrier, then since $a(t)$ is slowly varying compared to the carrier, the return consists of a delay $\tau = 2r(t)/c$ in $a(t)$ due to the over-all round trip

time from the radar to the satellite, whereas the fine range differences of the individual scatterers are manifested in the carrier component of the signal. Thus the returns from the two scatterers are,

$$\begin{aligned}s_1(t) &= a(t - \tau) \exp\{j\omega_0(t - 2r_1(t)/c)\} \\ s_2(t) &= a(t - \tau) \exp\{j\omega_0(t - 2r_2(t)/c)\}\end{aligned}$$

where c is the speed of electromagnetic propagation. Thus the measure of signal separation is,

$$\Delta(x) = \text{Re} \int_0^T a(t - \tau) a^*(t - \tau) \exp\left\{j \frac{2\omega_0}{c} (r_2(t) - r_1(t))\right\} dt \quad (8)$$

which from equations (6) and (7) is

$$\begin{aligned}\Delta(\theta) = \text{Re} \int_0^T & |a(t - \tau)|^2 \exp\left\{j \frac{2\omega_0(R^2 + \rho^2)^{1/2}}{c} \left[q \cos\left(\sigma - \frac{V}{R}t - \phi - \theta\right) + \right. \right. \\ & \left. \left. \frac{q^2}{4} \cos^2\left(\sigma - \frac{V}{R}t - \phi - \theta\right) - q \cos\left(\sigma - \frac{V}{R}t - \phi\right) - \right. \right. \\ & \left. \left. \frac{q^2}{4} \cos^2\left(\sigma - \frac{V}{R}t - \phi\right) \right] \right\} dt\end{aligned}$$

Thus, $x = \theta$ is the parameter that is to be resolved. This would provide a measure of the separation between the two scatterers and from which the lateral dimension of the satellite can be inferred. The resolution function describes the capability of measuring θ , on the basis of signals separation.

To obtain a more revealing form for the resolution function, first separate the angle expressions from the time expressions by expansion of some trigonometric identities.

$$\begin{aligned}\Delta(\theta) = \int_0^T & |a(t - \tau)|^2 \exp\left\{j \frac{2\omega_0(R^2 + \rho^2)^{1/2}}{c} \left[q(\cos(\sigma - \phi) - \cos(\sigma - \phi - \theta)) \cos \frac{V}{R}t \right. \right. \\ & + q(\sin(\sigma - \phi) - \sin(\sigma - \phi - \theta)) \sin \frac{V}{R}t + \\ & \left. \frac{q^2}{8} (\cos 2(\sigma - \phi) - \cos 2(\sigma - \phi - \theta)) \cos \frac{2V}{R}t \right. \\ & \left. + \frac{q^2}{8} (\sin 2(\sigma - \phi) - \sin 2(\sigma - \phi - \theta)) \sin \frac{2V}{R}t \right] \right\} dt\end{aligned}$$

Furthermore, the orbital parameters are typically, $\rho = 4000$ miles, $R = 500$ miles, $V = 5$ miles/sec., and thus $\frac{V}{R}t \ll 1$, and $\theta \ll 1$. There is also no loss in generality if $\sigma - \phi$ is set to zero. It then follows, after some mathematical manipulation, that the resolution function can be expressed as

$$\Delta(\theta) = \int_0^T |a(t - \tau)|^2 \exp\left\{j \frac{4\pi}{\lambda} \sqrt{1 + \left(\frac{\rho}{R}\right)^2} (q + q^2/4) V \theta t\right\} dt \quad (9)$$

where $\lambda = 2\pi c/\omega$ is the transmitted wavelength. It only remains to specify the modulation waveform so as to determine the resolvability of the scatterers separation angle, θ .

Consider a conventional pulse-doppler waveform as shown in Figure II; that is,

$$a(t) = \sum_{n=0}^N [u(t - nT_R) - u(t - nT_R - T_p)] \quad (10)$$

where T_p = pulse width, T_R = pulse period, and $N = T/T_R$ which is the number of pulses in the dwell time of T , and

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

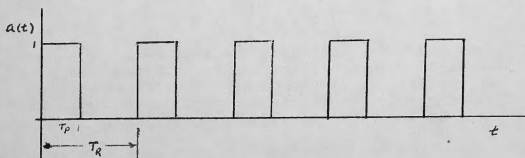


Figure II Transmitted Waveform

On substitution of this waveform into the resolution function of equation(9), and performing the indicated integration over the observation interval, yields

$$\Delta(\theta) = T_p \frac{\sin KT_p \theta}{K \theta T_p} \sum_{n=0}^N \cos nKT_R \theta$$

where $K = \frac{4\pi}{\lambda} \sqrt{1 + \left(\frac{c}{R}\right)^2} \quad (q + q^2/4)V$

which for the typical orbital parameters is in the order of 10^6 at x-Band.

The essentially $\sin x/x$ expression before the summation is a slowly varying function of θ . Thus for regions of interest where θ is in the order of micro-radians, the resolution is characterized by

$$d(\theta) = \sum_{n=0}^N \cos n T_R K \theta \quad (11)$$

The magnitude of this function is shown in Figure III. The width of the central peak describes the resolvability of the angle θ in terms of the separability of the two signals returned from the individual scatterers which are apart by relative to the earth center. From which, and the expression for K , it is observed that resolution of θ increases linearly with satellite velocity, as well as total observation time, T . Thus, for the representative satellite velocity of 5 miles/sec., the 3db widths of the resolution function's central peak are 1.48, 0.295, and 0.148 micro-radians for observation intervals of $T = 1$ second, 5 seconds, and 10 seconds respectively. For a satellite that is 500 miles above the earth, these correspond to resolving scatterers that are separated in the satellite's lateral dimension by 35 feet, 7 feet and 3.5 feet, respectively.

This demonstrates that the angle θ , is indeed, the parameter which was sought in the introduction, to provide a measure of satellite size, that would be resolvable entirely from a signals separation process. It remains only to consider the realization of such a process.

System Realization

In general, the radar return from the satellite will consist of an ensemble of signals from each of the total of say M scatterers. Therefore,

$$S(t) = \sum_{k=1}^M s(t, \theta_k)$$

where θ_k , represents the relative separations of the scatterers. The sequential development of the resolvability of these angles (equations (2), (6), (7), (8), (9), (11)), also suggests the technique for accomplishing the parameter resolution. Essentially, a correlation process is described. This is to say that if, the returned signal ensemble, $S(t)$, is correlated with a reference signal $s(t, \theta)$, for successive values of θ , where θ ranges over the expected angular dimension of the satellite, then peaks corresponding to the function $d(\theta_k)$ will occur at values $\theta = \theta_k$, $k=1, 2, \dots, M$. The relative positions of these peaks resolve the scatterers and serve as a measure of the satellite's lateral dimension. Thus, the scheme for realizing this resolution technique is outlined in Figure IV.

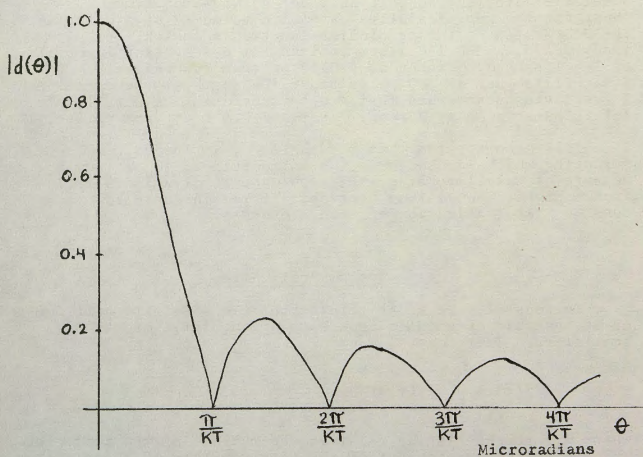


Figure III The Resolution Function

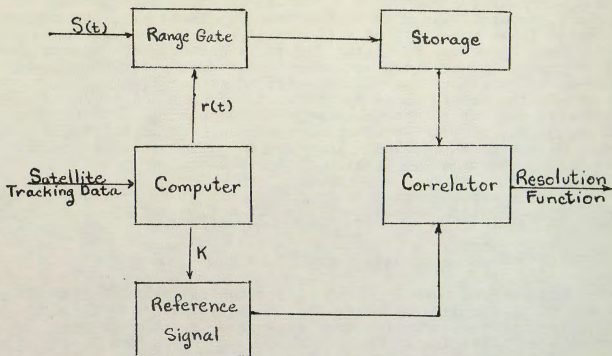


Figure IV

Signal Processing Scheme

Basically, the received radar signal ensemble, $S(t)$, is range gated, demodulated and suitably stored for subsequent correlations. A reference signal, $s(t) = a(t)\exp\{jK\theta t\}$, is locally generated for the interested range of values of θ . The stored received signal is read out and correlated with the reference for each value of θ . An auxiliary computer continuously updates the range gate $\tau = r(t)$, and computes K , which is a function of the satellite velocity, and position, for generating the reference signal, from tracking data. The correlation may be accomplished either electronically or optically. Electronically, the received signal can be stored in some form of scan-in, scan-out storage tube. The reference signal can be generated by a variable local oscillator of frequency $K\theta$. This is mixed with the received signal and envelope detected to obtain the resolution function outputs. Optically, the received signal is recorded on film. Correlation is achieved by suitably lens imaging the film with a reference signal mask. Both electronic and real-time optical correlators that can be employed in such a system scheme have been developed by various organizations but security measures prohibit any detailed discussion of them in this unclassified paper.

Those familiar with airborne ground-mapping processes perhaps will recognize a possibility of viewing this space object size measurement technique as an inverse analog to the ground-mapping "synthetic aperture" device. Whereas, in the ground-mapping system, the observation station is in controlled motion, the satellite measuring system is stationary, but the observed is in uncontrolled motion that must be measured or predicted. However, the parameter θ , which has been chosen is an invariant parameter relative to the geocentric coordinate system. Furthermore, satellite position or range, which only occurs in the resolution function as the factor $\sqrt{1 + \theta^2/R^2}$ indicates minimal effect from range deviations and thus further validates the circular orbit approximation from which the invariant angle parameter is derived. In addition, any errors in predicting the satellites velocity is also insignificant.

For example, if the predicted velocity V' is in δ percentage error from the true velocity V , then $V' = (1 + 100\delta)V$ and

$$d'(\theta) = \sum_{n=0}^N \cos nKTr(1 + 100\delta)\theta$$

Thus, whereas the zero crossing of the central peak of $d(\theta)$ is at $\theta = \pi/K$, the zero crossing for $d'(\theta)$ is at $\theta' = \pi/K(1 + 100\delta)$. By taking the zero-crossing as a measure of resolution the percentage difference is

$$\frac{\theta - \theta'}{\theta} = \frac{100\delta}{1 + 100\delta} \text{ percent} \quad (12)$$

Thus, for as much as a $\delta = 5\%$ error in satellite velocity measurement, this represents only a degradation of 4 inches in the 7 feet resolution capability of a 5 second observation time.

By similar argument the effect of the earth's rotational motion can be shown to be also negligible. The earth's rotational rate is approximately 0.03 miles/sec. This can be considered as a perturbation on a 5 miles/sec satellite velocity of about 0.6%. From equation (12) it represents only half an inch deviation in resolution for a 5 second observation interval.

Thus, analogous to the airborne ground mapping radar, the large aperture discussed in the introduction as required for resolving the individual scatterers on a space-object in angle, has indeed been synthesized in the signal domain. In fact, the otherwise infeasible demand on the physical antenna, is now completely reasonable. It need only provide a beam sufficiently wide to cover the distance traversed by the satellite during the observation interval. For a 5 second dwell and a 5 miles/sec. satellite, it only needs to have a 3° beam width.

Waveform Selection

A simple pulse-doppler waveform was selected to illustrate this resolution technique. Since the scatterers are closely spaced, the output of the satellite sizing system of Figure IV will consist of a sequence of $d(\theta_k)$ in close proximity. Thus, the near side-lobes of each as shown in Figure III, may be of sufficient height to be of mutual interference between consecutive main-lobe responses. A waveform whose resolution function consists of lower near-in side-lobes may be required. One technique of reducing these side-lobes is by a pulse-doppler waveform with a coded pulse period rather than the uniform one of the illustration. By this is meant a code that is representable by

$$a(t) = \sum_{n=0}^N [u(t - (n + \alpha_n)T_R) - u(t - (n + \alpha_n)T_R - T_p)]$$

where $\alpha_n < 1$, for $n = 1, 2, \dots, N$ is the code of nonuniform pulse periods. Substitution of this waveform into the resolution function of equation (9) results in

$$\begin{aligned} d(\alpha, \theta) &= \sum_{n=0}^N \cos KT_R(n + \alpha_n)\theta \\ &= \sum_{n=0}^N [\cos nKT_R\theta \cos(\alpha_n KT_R\theta) - \sin(nKT_R\theta) \sin \alpha_n KT_R\theta] \end{aligned}$$

Since $\alpha_n < 1$ and θ is small, this further reduces to

$$\begin{aligned} d(\alpha, \theta) &= \sum_n \cos(nKT_R\theta) - \sum_n KT_R\theta \alpha_n \sin(nKT_R\theta) \\ &= d(\theta) - \sum_n KT_R\theta \alpha_n \sin(nKT_R\theta) \end{aligned}$$

From which the code is found as

$$\alpha_n = \int_{\theta_1}^{\theta_2} \frac{d(\theta) - d(\alpha, \theta)}{\theta} \sin(nKT_R\theta) d\theta$$

for $n=1, 2, 3, \dots, N$. The coded waveform $\{\alpha_n\}$, is specified explicitly in terms of the required amount of side-lobe reduction from the resolution function of uniform pulse-period in any desired angle region of θ , $\theta_1 \leq \theta \leq \theta_2$; that is in terms of a specified amount of $d(\theta) - d(\alpha, \theta)$. The detailed synthesis of such waveforms which would be applicable to this space-object identification radar system scheme is found in reference (2).

Conclusion

The realization of a signal processing radar system which is capable of measuring the lateral dimension of a space object has thus been developed. The system is a signal processing technique developed in terms of an analytic angle resolution function formulated on the basis of a time-invariant geocentric scatterers separation angle. This angle was derived from a geocentric orbit geometry. The validity of the results of this coordinate reference was then established for contemporary artificial satellites. The angle function was derived from postulates which were also shown to be valid for the existing class of satellite parameters.

Moreover, the attribute of a resolution function based on a time invariant angle parameter provided certain desirable system characteristics. Resolution increases with length of correlation time and the time invariance makes long observation times possible. There are also no prohibitive requirements on the physical antenna. It is only necessary for the object to be within the radiated beam during the observation interval. Thus, only the correlation time and the object velocity dictate the antenna beamwidth. Other characteristics of the system are that resolution varies inversely with wave length and directly with object velocity. Furthermore, a priori object velocity estimation errors or effects due to the earth's rotational motion contribute very little degradation to resolution.

Bibliography

- (1) "Space Log", Vol 4 No.3, 1964, TRW Space Technology Laboratories, Redondo Beach, California
- (2) Wan, L.A.; "PPM Code for the Improvement of Doppler Shift Measurement", IEEE Transactions on Information Theory, Vol. IT-10 No.3, July, 1964